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✓ Spreadsheet
✓ DTS

MEMORANDUM FOR PRR (Contractor/In-House Publication)

2 June1999

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SUBJECT: Authorization for Release of Technical Information, Control Number: AFRL-PR-ED-TP-FY99-0124
T.C. Miller, "Overdeterministic Fracture Analysis and Singular Value Decomposition"
SEM Conference slides/paper approved 25 Apr 99 **(Public Release)**

Overdeterministic Fracture Analysis and Singular Value Decomposition

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Ravinder Chona
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SEM Spring Conference
Cincinnati, Ohio
June 1999

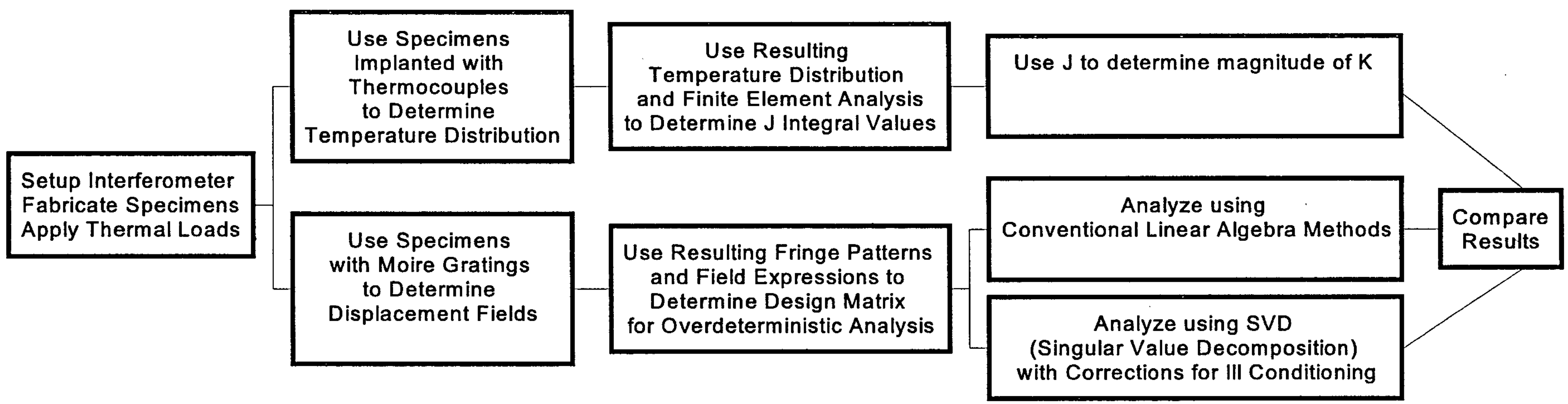
Outline of Presentation

In Bimaterial Fracture Problems, Singular Value Decomposition Can Be Used to Improve Results for Stress Intensity Factor Calculations

- Experimental Procedures Used
- Causes of Ill Conditioning
- Use of Singular Value Decomposition
- Comparison of Results
- Conclusions

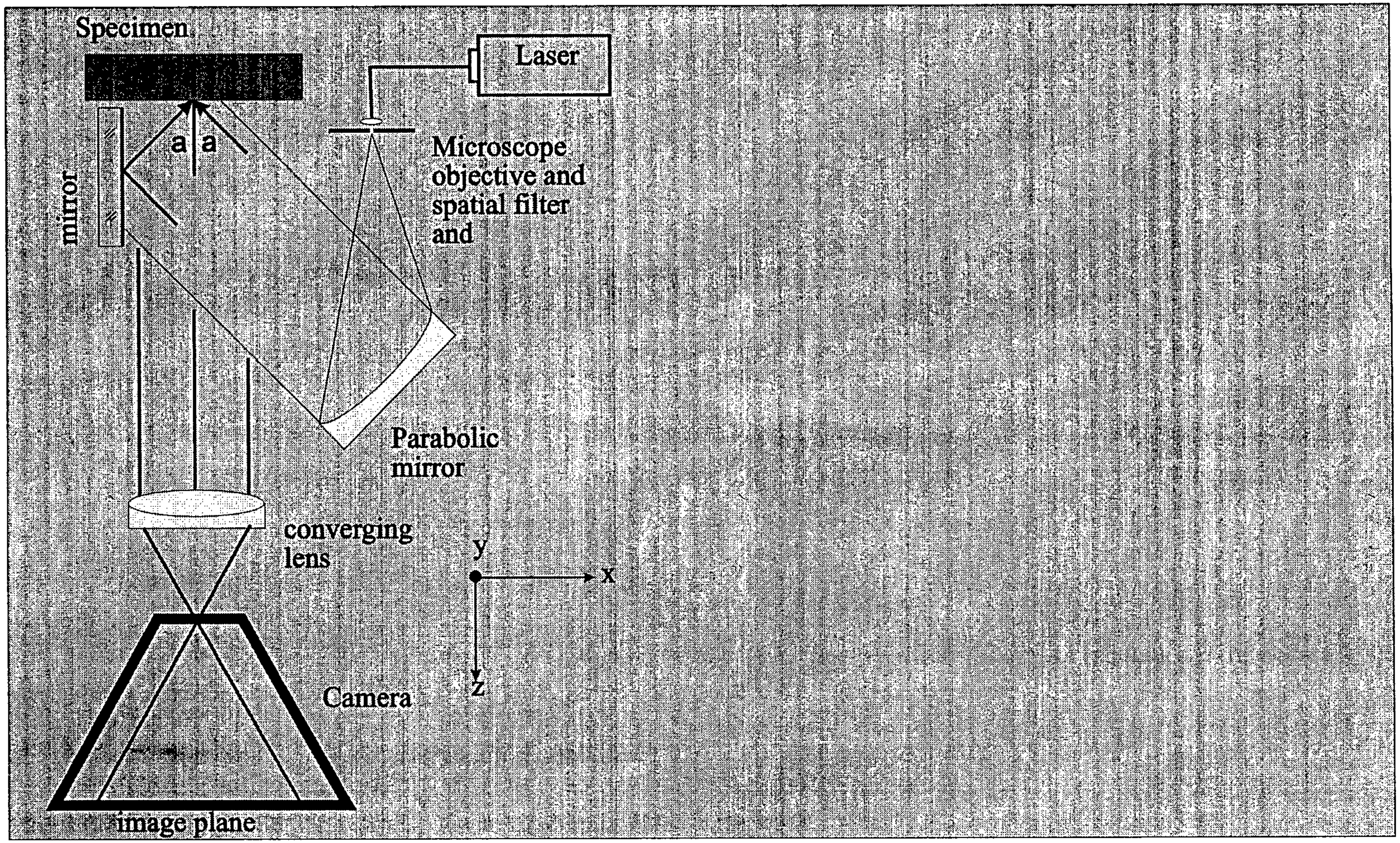
Outline of Experimental Methodology

Magnitude of Complex Stress Intensity Factors Were Arrived at By Three Different Means



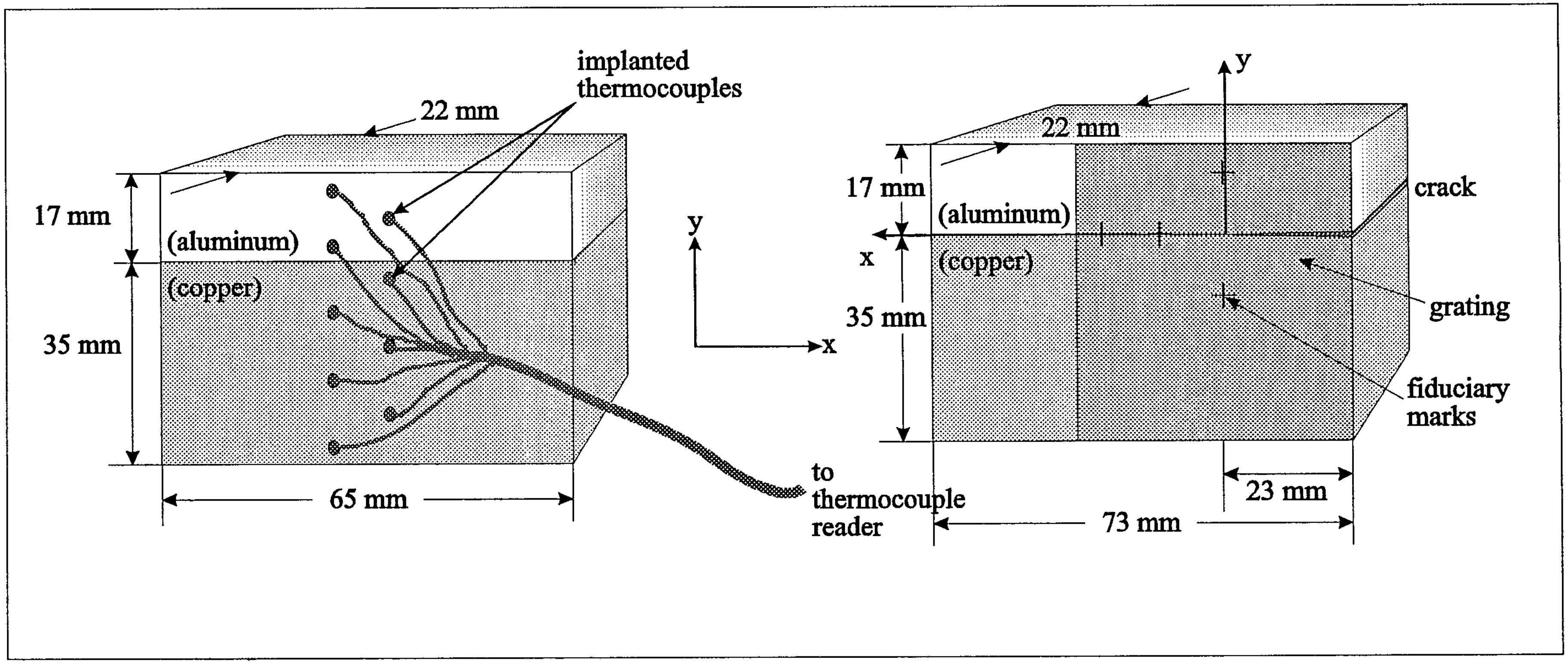
Moire Interferometer Setup

Simple Interferometer is Used to Capture U Displacement Fields



Specimen Fabrication for Aluminum- Copper Specimen

Shown are Specimens used with Thermal and Displacement Field Analyses



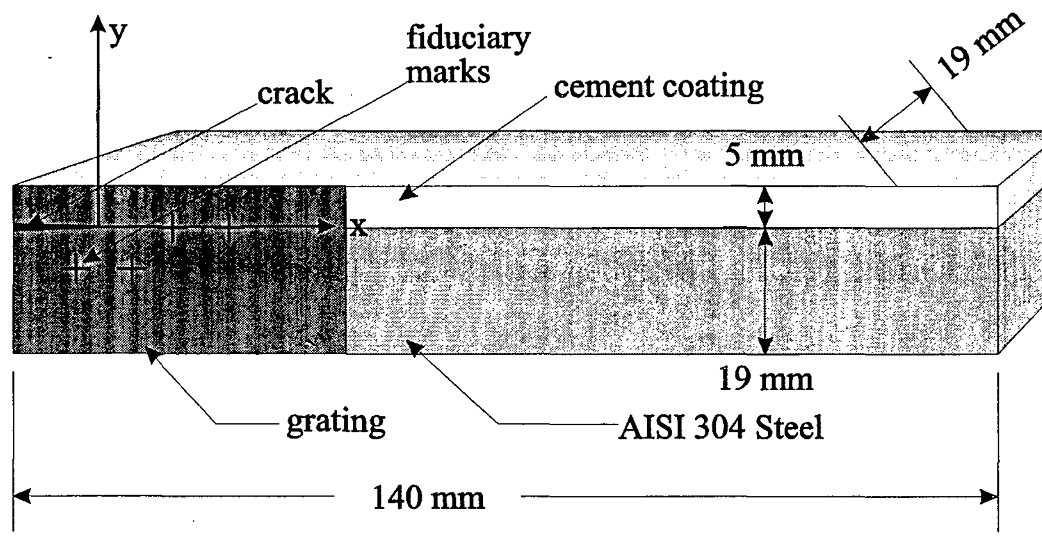
Mechanical and Thermal Properties for the Aluminum-Copper Specimen

Property	Aluminum	Copper
Young's modulus [GPa]	71.7	120.0
Poisson's ratio	0.34	0.33
density [kg/m^3]	2770.0	8800.0
coefficient of thermal expansion [$10^{-6}/K$]	24.0	17.0
thermal conductivity [$W/m^2 K$]	270.0	441.6
specific heat [J/kg K]	875.0	420.0

Specimen Fabrication for Steel-Thermocouple Cement Specimen

Highly Dissimilar Materials and Nearby Free Surface Effects are Incorporated

(A Similar Specimen was Constructed for the Specimen with Implanted Thermocouples)

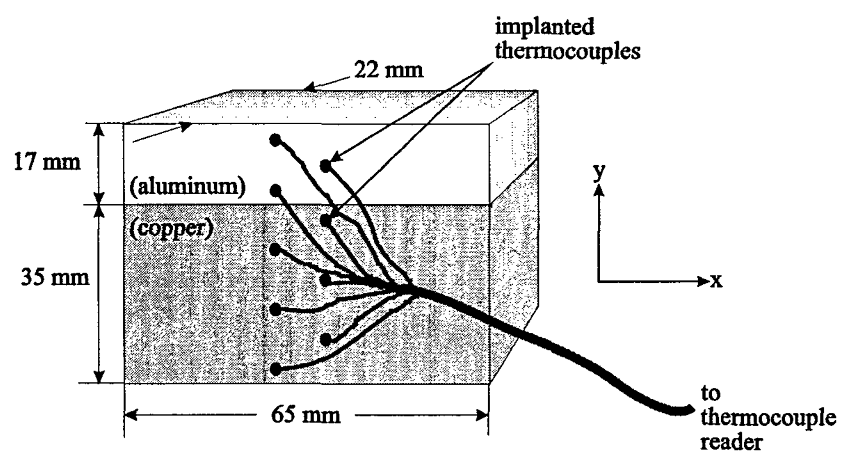


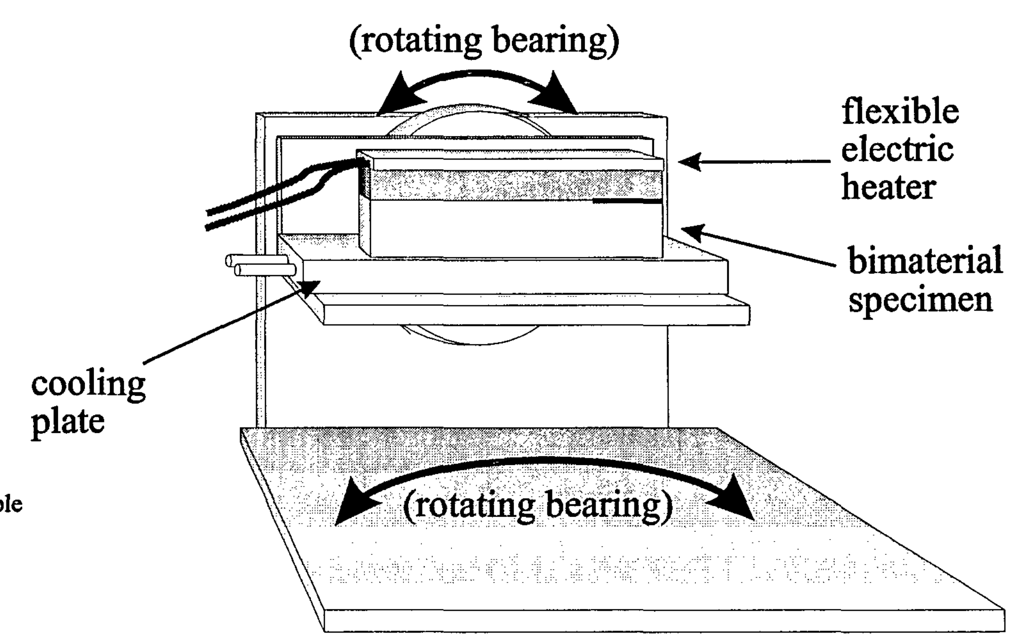
Mechanical and Thermal Properties for the Steel-TCC Cement Specimen

Property	AISI 304 Steel	TC cement
Young's Modulus [GPa]	218	3.24
Poisson's ratio	0.29	0.30
density [kg/m^3]	7834	3173
coefficient of thermal expansion [10^{-6}/K]	17.2	19.7
thermal conductivity [W/m^2 K]	16.2	1.15
specific heat [J/kg K]	500	100

Application of Thermal Loads to Specimens

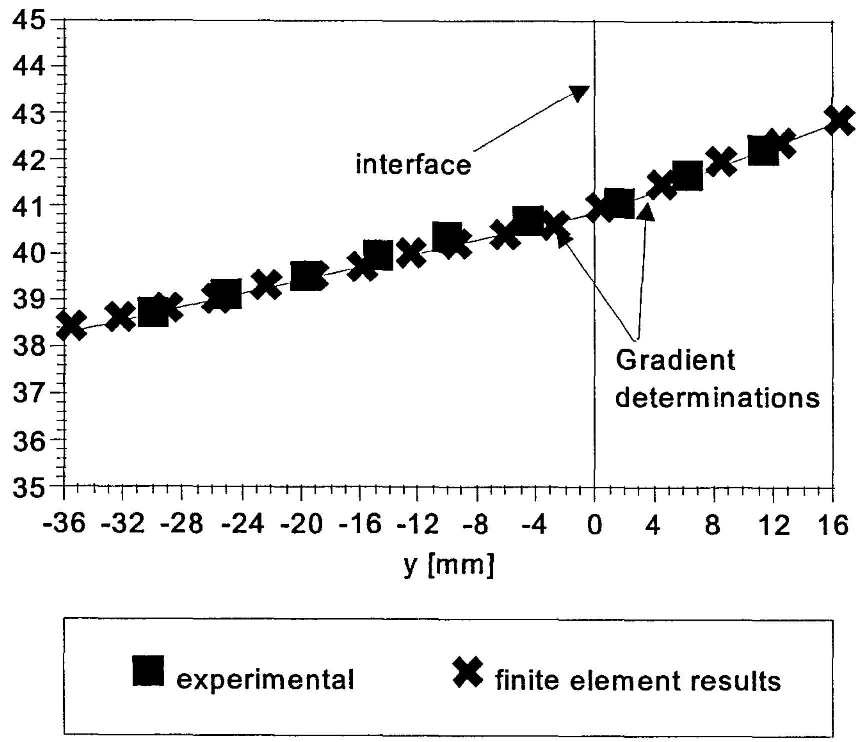
Applying Heating to Top of Specimen and Cooling to Bottom Provides One-Dimensional Temperature Fields (Except Near the Crack Tip)



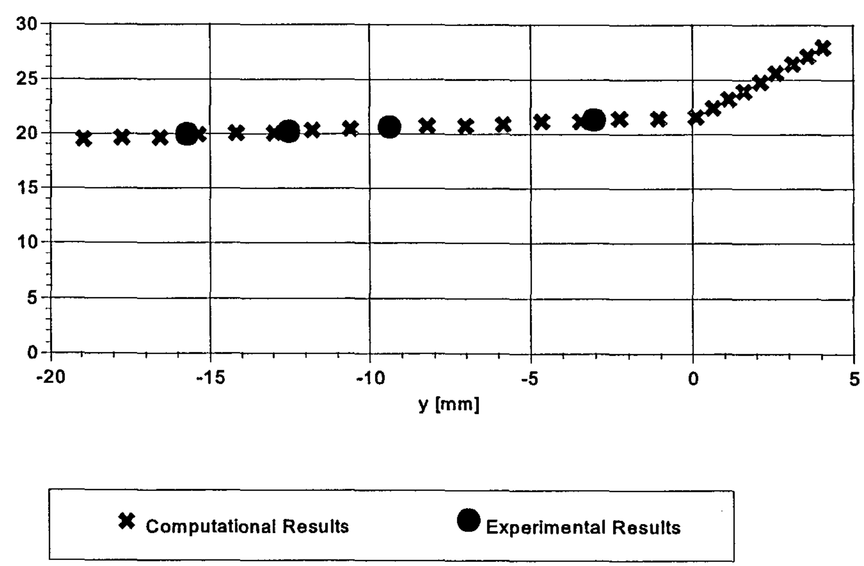


Determination of Temperature Distributions from Thermocouple Data

Thermal Boundary Conditions Can be Simulated Using Finite Element Methods



Aluminum-Copper Specimen

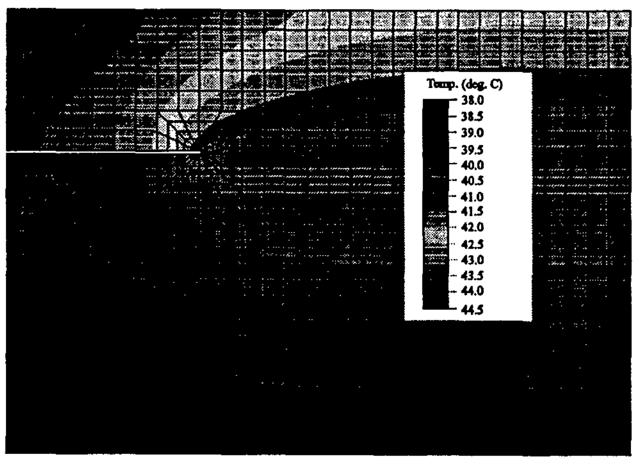


Steel-TCC Specimen

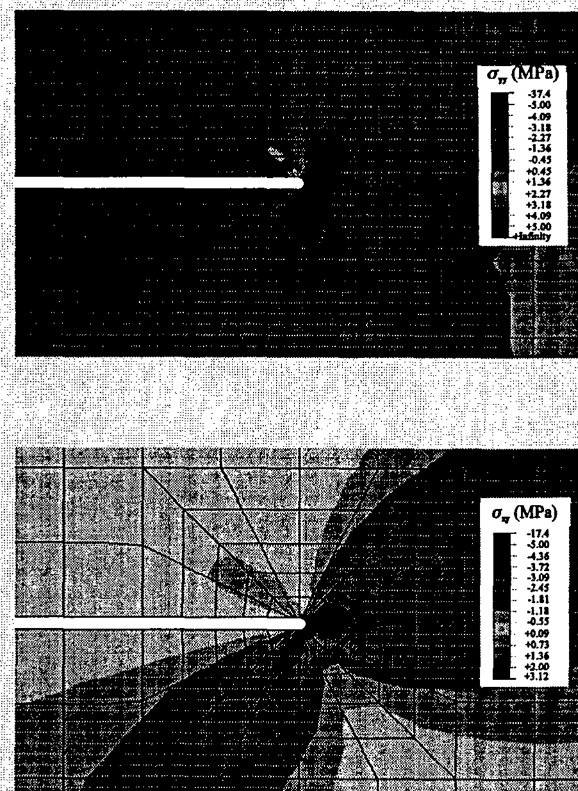
Thermal Loads and Numerical Modeling Gives Magnitude of |K|

Finite Element Results Give Stress Fields, J Integral, and Magnitude of Complex Stress Intensity Factor

Temperature Contours

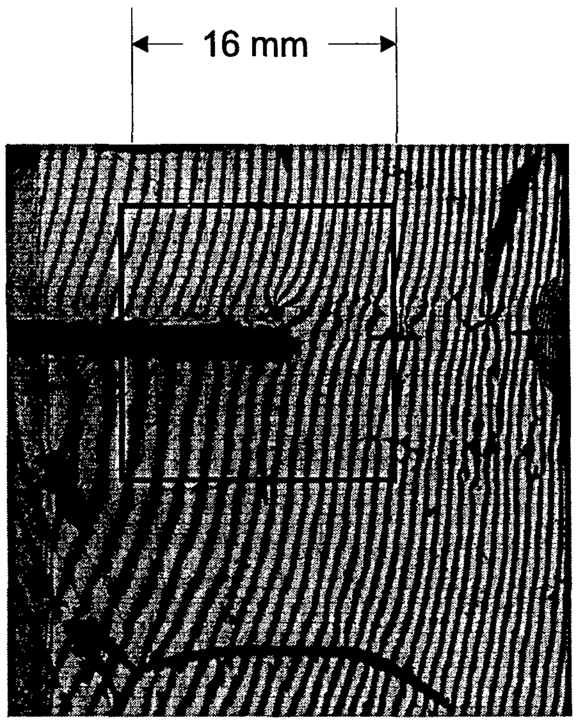


Stress Contours

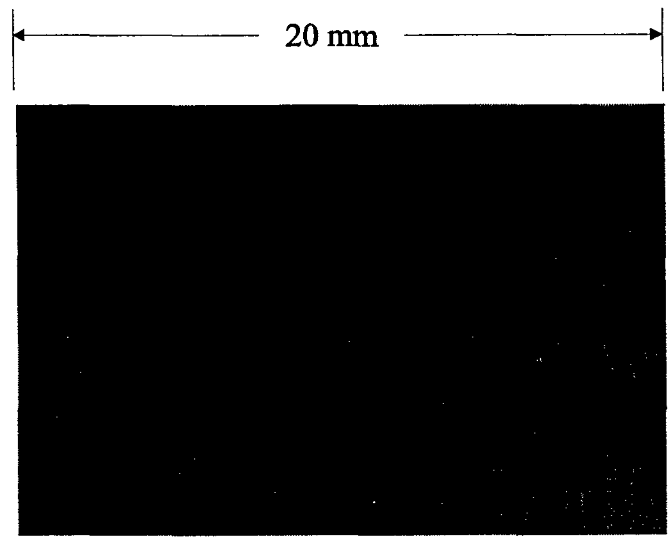


Moiré Interferometry is Used to Obtain Fringe Patterns for Subsequent Analysis

Fringe Patterns Are Digitized and Data is Used in Overdeterministic Analysis



Aluminum-Copper Specimen



Steel-TCC Specimen

Causes of Unstable Solutions

In some cases the normal equations work adequately, but in other cases a very ill-conditioned matrix occurs, and the result is fitted parameters with very large magnitudes that are unstably balanced to cancel out almost precisely when the fitted function is evaluated.

This occurs most often because the data do not clearly distinguish between two or more of the basis functions provided. The result is a large set of ambiguous solutions exist.

In difficult problems, the ambiguities may be hard to discern.

Displacement Field Equations for Homogeneous Materials

Overall Field Equation is a Combination of Arbitrary Translation/Rotation and Near Tip Deformations

$$u_y = Px + Qy + R$$

$$u_x = \frac{1}{E}\Big[\sum_{j=0}^{\infty} C_{2j}\frac{r^{j+1/2}}{j+1/2}\big((1-v)\cos(j+1/2)\theta - (1+v)(j+1/2)\sin\theta\,\sin(j-1/2)\theta\big)$$
$$+\sum_{j=0}^{\infty} C_{2j+1}\frac{r^{j+1}}{j+1}\big(2\cos(j+1)\theta - (1+v)(j+1)\sin\theta\sin(j\theta)\big)\Big]$$

$$N(r,\theta) = L(r,\theta) + M(r,\theta)$$

Displacement Field Equations for Bimaterial Problems

$$L(x,y) = P_1 x + R \qquad y \geq 0$$

$$P_2 x + R \qquad y < 0$$

$$M(x,y) = \frac{1}{2\mu_1}[a_{0r} r^{1/2}(f_{0r})_1 - a_{0j} r^{1/2}(f_{0j})_1 + b_{0r} r(g_{0r})_1 - b_{0j} r(g_{0j})_1] \quad y \geq 0$$

$$= \frac{1}{2\mu_2}[a_{0r} r^{1/2}(f_{0r})_2 - a_{0j} r^{1/2}(f_{0j})_2 + b_{0r} r(g_{0r})_2 - b_{0j} r(g_{0j})_2] \quad y < 0$$

$$N(r,\theta) = L(r,\theta) + M(r,\theta)$$

Solution of Linear Algebra Problems Using Conventional Methods

Obtain Displacement Data from Moire Interferometer Setup

Formulate Linear Algebra Problem Using Displacmement Data and Combined Field Expressions

Use Linear Algebra to Transform Rectangular Design Matrix into a Square Matrix

Solve Linear Algebra Problem to Determine Unknown Coefficients in Filed Expression (and Complex Stress Intensity Factor)

Solution of Linear Algebra Problems Using Conventional Methods

$$\underset{m \times 1}{\{N\}} = \underset{m \times n}{[f]} \; \underset{n \times 1}{\{C\}}$$

$$[f]^T \{N\} = [f]^T [f] \{C\}$$

$$Let \quad \{d\} = [f]^T\{N\}, \quad [a] = [f]^T[f]$$

$$\underset{n \times 1}{\{d\}} = \underset{n \times n}{[a]}\underset{n \times 1}{\{C\}}$$

The Use of the SVD Method to Solve Overdeterministic Problems

Condition of Matrix is Determined and Adjusted for, if Necessary

- Obtain Displacement Data from Moire Interferometer Setup
- Formulate Linear Algebra Problem Using Displacmement Data and Combined Field Expressions
- Decompose Rectangular Design Matrix into [U], [V], and [W] Matrices
- Check [W] Matrix for Presence of Ill-Conditioning
- Correct for Ill-Conditioning, if Necessary
- Solve Linear Algebra Problem to Determine Unknown Coefficients in Filed Expression (and Complex Stress Intensity Factor)

How the SVD Method Corrects for Ill-Conditioning

$$\{N\} = [f]\{C\} = \underset{m x n}{[U]}\,\underset{n x n}{[w]}\,\underset{n x n}{[V]^T}\,\{C\}$$

Large condition numbers indicate ill-conditioning. When the reciprocal of the condition number approaches the computational precision, the presence of ill-conditioning will significantly affect solution accuracy. If this occurs, the adverse effects can be minimized by setting the terms $1/w_j$ in equal to zero for all sufficiently small w_j.

The effect of ill-conditioning is to produce an infinite set of solutions that all approximately solve the linear equation $\{N\} = [f]\{C\}$. Zeroing these diagonal elements selects from this set the solution the one that minimizes the residual $R = |\ \{N\} - [f]\{C\}\ |$. The result is that the SVD method with correction is often better than both direct methods and uncorrected SVD methods, as is shown by the experimental results below.

Use of the SVD Method to Solve Linear Algebra Problems

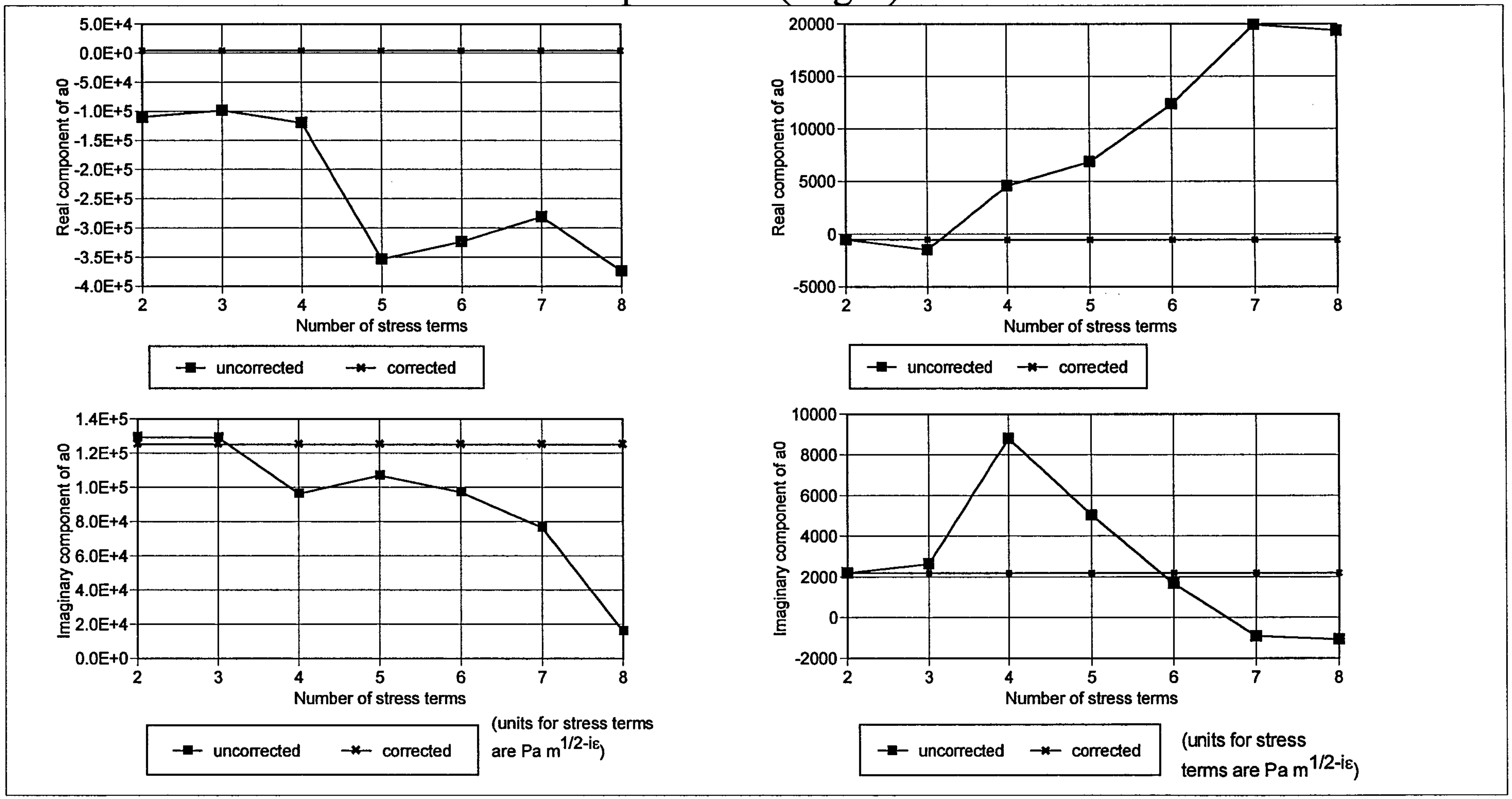
$$\underset{m \times 1}{\{N\}} = \underset{m \times n}{[f]}\ \underset{n \times 1}{\{C\}}$$

$$\{N\} = [f]\{C\} = \underset{m \times n}{[U]}\,\underset{n \times n}{[w]}\,\underset{n \times n}{[V]^T}\{C\}$$

$$\{C\} = [V] \cdot [diag(1/w_j)] \cdot (U^T \cdot \{N\})$$

Correcting for Ill-Conditioning Using SVD Gives Improved and More Stable Results

Shown Are Results for Aluminum-Copper Specimen (Left) and Steel-TCC Specimen (Right)



Results for Aluminum-Copper Bimaterial Specimen

Parameter	Trial 1	Trial 2	Average
P_1	375.8×10^{-6}	395.6×10^{-6}	385.7×10^{-6}
P_2	338.6×10^{-6}	346.5×10^{-6}	342.6×10^{-6}
a_{or} [Pa $m^{1/2-i\epsilon}$]	4523	4920	4721
a_{oj} [Pa $m^{1/2-i\epsilon}$]	125200	118300	121700
$\|K'\|$ [Mpa $m^{1/2}$] (defined at 1 mm)	0.630	0.596	0.613
phase of K' [degrees] (defined at 1 mm)	-87.9	-87.6	-87.8
Related measurements			
Measurement	Source	Value	Related parameter
$(\varepsilon_{xx})_1$	Fringe pattern plots (see Fig. 37)	394.0×10^{-6}	P_1
$(\varepsilon_{xx})_2$	Fringe pattern plots (see Fig. 37)	279.0×10^{-6}	P_2
$\|K'\|$ [Mpa $m^{1/2}$] (defined at 1 mm)	Finite element J integral calculation	0.601	$\|K'\|$ or $\|a_0\|$ (above)

Results for Steel-TCC Bimaterial Specimen

<table>
<tr><th>Parameter</th><th colspan="3">Experimental Value</th></tr>
<tr><td>a_{0r} [Pa $m^{1/2-i\epsilon}$]</td><td colspan="3">-526</td></tr>
<tr><td>a_{0j} [Pa $m^{1/2-i\epsilon}$]</td><td colspan="3">2203</td></tr>
<tr><td>$|K'|$ [Mpa $m^{1/2}$] (defined at 1 mm)</td><td colspan="3">12.1</td></tr>
<tr><td>phase of K' [degrees] (defined at 1 mm)</td><td colspan="3">31.8</td></tr>
<tr><td colspan="4">Related measurements</td></tr>
<tr><td>Measurement</td><td>Source</td><td>Value</td><td>Related parameter</td></tr>
<tr><td>$|K'|$ [Mpa $m^{1/2}$] (defined at 1 mm)</td><td>Finite element J integral calculation</td><td>11.1</td><td>$|K'|$ or $|a_0|$ (above)</td></tr>
</table>

Comparison of Experimental and Numerical Results

Use of Local Collocation Method with SVD Gives Good Agreement with Numerical Computations

Material Pair	SVD-Local Collocation Method	Finite Element Results
aluminum-copper	0.613	0.601
steel-thermocouple cement	12.1	11.1

(Units are MPa $m^{1/2}$)

Conclusions

- The Presence of a Nonzero Bimaterial Parameter in Interfacial Fracture Problems Can Cause Problems When Overdeterministic Methods Are Employed.

- One Means to Remedy the Problem of Ill-Conditioning is to use Singular Value Decomposition to Solve the Linear Algebra Problem.